Solvay Brussels School Economics & Management
Université Libre de Bruxelles
2011

ULB

FINANCIAL RISK MANAGEMENT & GOVERNANCE
GEST-D-437
Prof. H. Pirotte

LASTNAME :

STUDENT ID :

FIRSTNAME :

Final Exam

1st session

Monday 30 May 2011 – aud. H.1308

Indications

Please follow these indications:

1. The exam lasts 2.5 hours in total but was designed to be answered in 2 hours.
2. Please verify that your document contains exactly 12 pages.
3. You are allowed to come with a 5-page recto-verso handwritten cheat sheet, i.e. 10 pages in total.
4. Each question is worth one point unless otherwise stated, i.e. there are 26 points in total, including two "bonus" (or facultative) questions of 3 points each.
5. Please write your first name and last name on the first page.
6. Good work!!!

Problems

P1 Short questions

Q1 The gamma and vega of a delta-neutral portfolio are 50 (per € per €) and 25 (€ per %) respectively. Estimate what happens to the value of this portfolio when there is a shock to its market causing the underlying asset price to decrease by €3 and its volatility to increase by 4%.

Q2 A corporate treasury has been borrowing a substantial amount of money on a short-term basis, i.e. one-month borrowings rollovered automatically* for 5 years. Payments are based on the 1-month LIBOR. The chief treasurer would like to swap this amount to a fix rate by using a swap. Knowing that the 0-coupon curve presents the following shape, how much do you expect the swap rate to be on such a case?

*"automatically" means that, as long as the company is not hitting a given interest coverage ratio, the bank will automatically renew the contract for one more month. It is therefore a close monitoring from the bank.

Term (years)	0-coupon rates
1	1.5844%
2	1.5236%
3	1.8990%
4	2.2520
5	2.5551

Q3 In the previous problem, and considering both the original borrowing contract and the swap, what is your view on the new duration of the financing? What does this duration mean exactly to you? How is the security of funding impacted by the existence of the swap?

Q4 Calcon Inc. has in its portfolio a 3-year structured product that comprises a FRN product (annual payments, so the reference rate is the 1y rate) bundled with a floor at 3% and a cap at 6%. Currently the 0-coupon curve is:

Term (years)	0-coupon rates
1	1%
2	1.5%
3	2.5%

The volatility of the 1y rate is 25%. The counterparty announces you that the price they would be willing to buy back this product for is 96% (of the nominal amount invested). What do you think about this price?

Q5 The regulatory report of Truong Bank states that the quarterly VaR of this bank is VND 100 billion at a 95% confidence level. What is the proper interpretation of this statement for an annual horizon?

Q6 Given the following 30 ordered percentage returns of an asset, calculate the VaR and expected shortfall at a 90% confidence level: -16, -14, -10, -7, -7, -5, -4, -4, -4, -3, -1, -1, 0, 0, 0, 1, 2, 2, 4, 6, 7, 8, 9, 11, 12, 12, 14, 18, 21, 23. (source: FRM exam)

Q7 Suppose that the change in the value of a portfolio over a 1-day time period is normal with a mean of zero and a standard deviation of $2.5 million. What is the 5-day 97.5% VaR if there is first-order daily autocorrelation with correlation parameter equal to 0.2?

P2 CreditVaR

Loannie Bank would like you to assess the value of their portfolio of loans. They have currently a portfolio of 20 BB-average borrowers. Following some rating agency' ratings transition matrix, the probabilities of transitions are the following ones for a BB after one year:

	One year after							
Today	AAA	AA	A	BBB	BB	B	C	Default
BB	0%	1%	2%	6%	80%	5%	4%	3%

Given current estimations of commonality in the default risk of companies and implied numbers from CDO contracts, studies show a base correlation between this type of borrowers of 50%. The LGD based on the collection experience of the bank in the case of defaults is estimated around 70%.

Q8 How much can you assess the potential probability of default to be on this entire portfolio, taking a 99% confidence level into consideration?

Q9 Based on your previous answer (thus under the same confidence level), Loannie Bank would like to know how high are the chances to have no more than 10 defaults in the portfolio of 20 loans.

Remark : if you didn't find any answer to the previous question, please use a probability of 30%.

Q10 Loannie Bank would like to create securities out of this portfolio of loans. More precisely, Loannie Bank would like to create a risky (in terms of credit risk) and a quasi-riskfree security on that portfolio. As in a CDO, the risky security is assumed to take the losses "before" the quasi-riskfree one, so that the latter is indeed "quasi-riskfree" (quasi because you would define this security as potentially riskfree given the statistical parameters you know from the past; there is no guarantee). How would you define/design those two securities based on the portfolio of 20 loans, given what you know today?

Remark : if you didn't find any answer to question Q8, please use a probability of 30%.

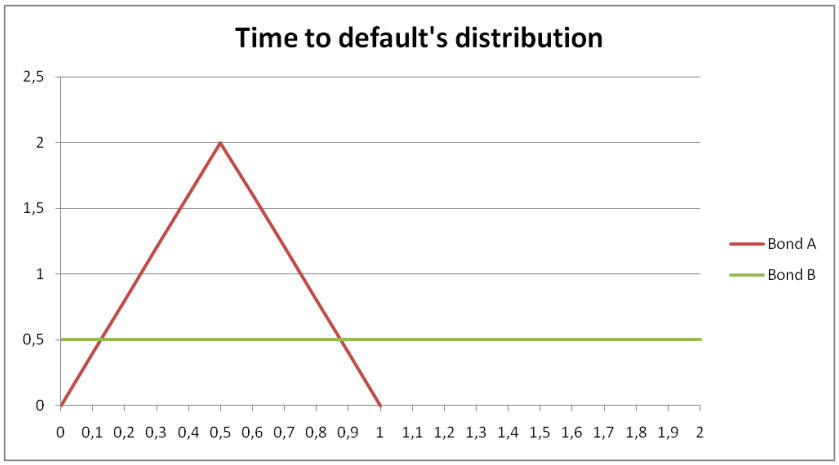
P3 Volatilities and dependencies…

Q11 Why is the study of dependencies so fashionable in risk management? Develop. No trivial comments accepted.

Q12 Suppose that the silver spot price closed yesterday at €28 and its volatility was estimated at 2% per day. The today's closing price is €25. Please update your volatility estimation in the context of the EWMA model with λ=0.94.

Q13 Do the same as in question Q12 but assuming this time a GARCH(1,1) model with $\omega = 0.000006$, $\alpha = 0.02$ and $\beta = 0.92$.

Q14 You have two bonds in your portfolio, Bond A and Bond B. You know that bond A will default within one year and Bond B will default before the end of year 2. The marginal distributions of time to default are shown in the graph below. Can you compute the probability that both bonds will default before 0,9 year using the minimum copula?



P4 Interest-rate risk measurement

In the table below, you will find data from the risk management department concerning the change in value of your bond portfolio for some moves of the yield curve (in bp).

Year	Move (bp)	Change (million €)
1	3	-0.84
2	5	-0.6
3	2	0.1
5	2	0.5
7	1	0.3
10	1	-0.2

Q15 Can you find the values of partial durations for those years (the total value of your portfolio is € 10 million)?

Q16 What's the total duration of your portfolio (you invest in bonds with maturities less than 10 years)?

Q17 If you consider that there is only two possible movements for the yield curve (i.e.parallel shift and bowing) which one will affect you most?

P5 In the news...

Q18 Please read the following press article, extracted from the Financial Times, December 20, 2011. Comment on the lines of liquidity vs. solvency criteria. (**3 points**)

Basel liquidity rules, going neo-medieval

Posted by **Joseph Cotterill** on Dec 20 16:47.

Can we talk a bit more about the scandal of Basel III allowing banks to give government bonds a zero risk weighting on their books? This time regarding Basel's liquidity rules.

Actually, can we talk about the related global shortage of AAA-rated assets and what *that* means for sovereign debt as well?

Buried in recent regulations on Basel's liquidity coverage ratio, we've found a few interesting new provisions on what are called Level 1 and Level 2 liquid assets. Banks have to hold enough of these to be able to withstand 30 days of net cash outflows under a stress scenario (think Lehman-level stress, bank runs, general end-times, etc).

The provisions basically present a 'post-sovereign' view of acceptable assets, if you will, worthy of contrasting with those zero risk weights.

Of Level 1 and Level 2

Now, Level 1 includes the questionably zero risk-weighted government bonds (alongside cash or central bank reserves, for example), and even allows banks to hold non-zero risk-weighted government bonds, so long as the sovereign in question is their domestic one. Let's use an example ripped from the headlines. The zero risk weighting will disappear if the sovereign's credit ratings fall below AA-. This is a situation Ireland now finds itself in: but Irish banks can still zero risk weight its bonds in their books, because Ireland's their domicile. Madness, we tell you.

Anyway, you can have as many Level 1 assets in your LCR as you like, but for Level 2 assets — riskier govvies, corporate or covered bonds — your portfolio of this stuff gets a haircut of 15 per cent, and can thence only constitute up to 40 per cent of your LCR assets.

Clear? OK. Now imagine you're an Australian bank.

In the land of low sovereign debt...

There's not a lot of level 1 Australian government debt floating around. Simply because, Australia is in the lucky position of not having to issue much debt given its fiscal health. Same with Denmark, Canada, Norway and so on. Reuters noted this point on Thursday.

If you're a bank from one of these countries, you suddenly find yourself penalised in your search for enough high-quality liquid assets. A sign of much wider asset shortages in the market, we would add. Repo markets have been signalling for a while that they face a shortage of good quality collateral, given demand created by the rise of secured lending since the Lehman crisis — see developments here, here and here, for instance.

In the words of Basel III (emphasis ours):

Some jurisdictions may have an **insufficient supply of Level 1 assets** in their domestic currency to meet the aggregate demand of banks with significant exposures in this currency. In addition, in several of these currencies, the supply of Level 2 assets may be very limited. To address this situation, the Committee has agreed to develop alternative treatments for the holdings in the stock of liquid assets. This treatment will apply to very few jurisdictions and currencies where insufficiencies of liquid assets exist…

Alternative treatments? Why not just load up on other governments' debt instead? As you might ask. This does happen to be one of those options under consideration.

However, Basel's problem is that currency swap risk comes with foreign-currency bond holdings, which might be too hard to manage during the aforementioned stress scenario. Of course, this might provide the very reason that

FX and interest-rate swap markets are growing strongly; to keep up with banks' demand for turning (for example) UK gilts into Australian government bonds.

Still, there are other limits. We'd add that banks should be very careful what *eurozone* government's bond they might buy, particularly after, say, 2013 when that new bailout-n-haircut ESM becomes operational, creating a two-tiered bond market in Europe.

Thus Basel is also considering another option of allowing banks to open special facilities with their central banks which function only to provide assets for the LCR. This looks very much like Basel's unique version of those phantom securities that central banks already sometimes use for unblocking repo markets — but again, it'd be far from ideal to introduce too many phantom securities to the small Australian bond market (for example's sake once again).

The final option that's on the table is quite a bit more intriguing yet. As the Basel committee explains:

Option 3 – Additional use of Level 2 assets with a higher haircut: This option addresses currencies for which there are insufficient Level 1 assets, as determined by the prescriptive threshold and criteria, but where there are sufficient Level 2 assets. In this case, supervisors may choose to allow banks that evidence a shortfall of liquid assets in the domestic currency (to match the currency of the liquidity risk incurred) to hold additional Level 2 assets in the stock, up to a prescriptive limit to be determined by the Committee. These additional assets would have a higher haircut than the Level 2 assets that are included in the 40% cap.

More corporate bonds, more covered bonds.

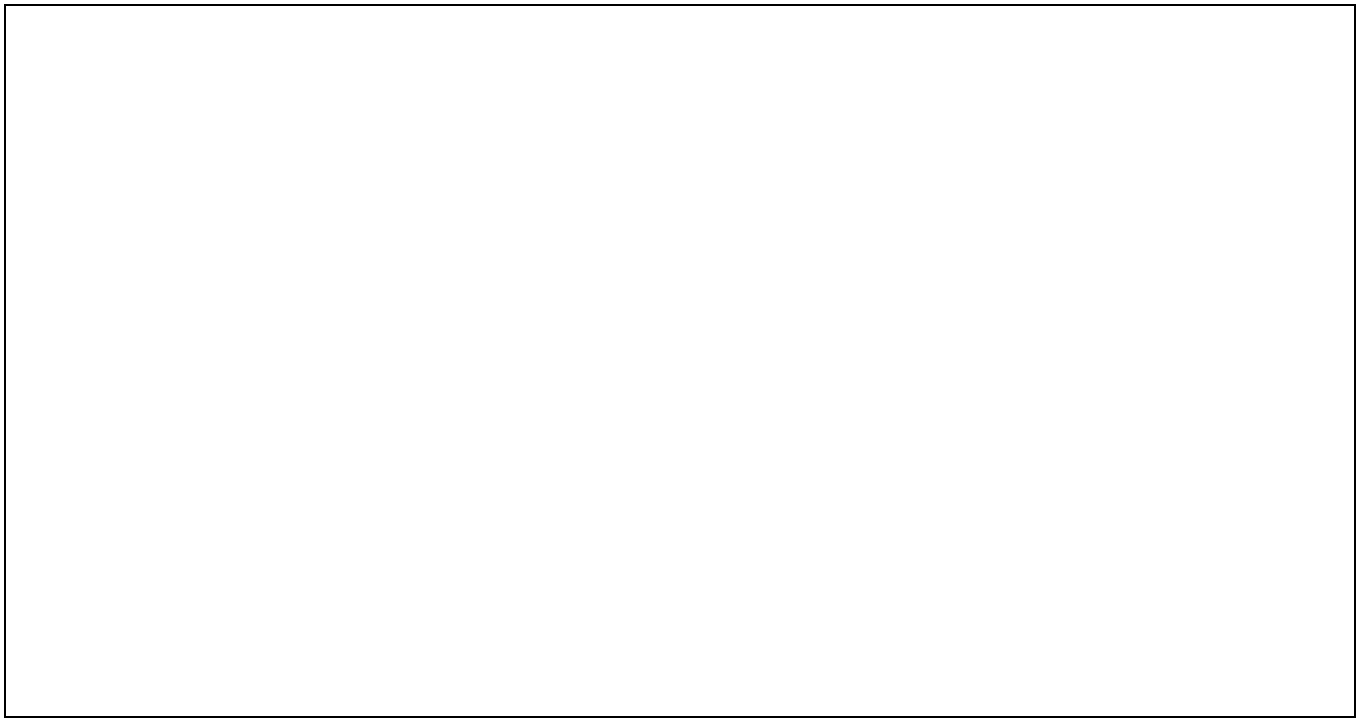
This would present a difficult situation for banks to be sure. They'd be asked to take on exposure to assets they might not want otherwise, in return for regulatory certainty.

Then there are the implications for the asset classes concerned. Take corporate bonds, for example. As per the Level 2 criteria, these can't have ratings below AA- or be issued by banks. That's not an entirely massive pool of corporate assets. The situation with covered bonds is complicated in another way. While covered bonds are widely used as funding instruments in Europe and have a pretty spotless record on defaults, Basel III has ticked off the industry once before.

Moreover, as a friendly investment bank strategist pointed out to us recently, covered bonds have a curious future as an asset class. They remain comprised of public sector or mortgage loans rather than say private-sector ones. Plus, the debate on whether regulators should be allowed to bail-in banks' senior debt could yet ensnare covered bonds.

All told, it's a strange regulatory environment for liquid assets once government bonds are removed, as this natural experiment of the problems of Australian or Danish banks demonstrates.

Bizarre — but quite possibly the wave of the future.



N(x) & N(-x)=1-N(x)

	0.000	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095
0.0	0.5000	0.5020	0.5040	0.5060	0.5080	0.5100	0.5120	0.5140	0.5160	0.5179	0.5199	0.5219	0.5239	0.5259	0.5279	0.5299	0.5319	0.5339	0.5359	0.5378
0.1	0.5398	0.5418	0.5438	0.5458	0.5478	0.5497	0.5517	0.5537	0.5557	0.5576	0.5596	0.5616	0.5636	0.5655	0.5675	0.5695	0.5714	0.5734	0.5753	0.5773
0.2	0.5793	0.5812	0.5832	0.5851	0.5871	0.5890	0.5910	0.5929	0.5948	0.5968	0.5987	0.6006	0.6026	0.6045	0.6064	0.6083	0.6103	0.6122	0.6141	0.6160
0.3	0.6179	0.6198	0.6217	0.6236	0.6255	0.6274	0.6293	0.6312	0.6331	0.6350	0.6368	0.6387	0.6406	0.6424	0.6443	0.6462	0.6480	0.6499	0.6517	0.6536
0.4	0.6554	0.6573	0.6591	0.6609	0.6628	0.6646	0.6664	0.6682	0.6700	0.6718	0.6736	0.6754	0.6772	0.6790	0.6808	0.6826	0.6844	0.6862	0.6879	0.6897
0.5	0.6915	0.6932	0.6950	0.6967	0.6985	0.7002	0.7019	0.7037	0.7054	0.7071	0.7088	0.7106	0.7123	0.7140	0.7157	0.7174	0.7190	0.7207	0.7224	0.7241
0.6	0.7257	0.7274	0.7291	0.7307	0.7324	0.7340	0.7357	0.7373	0.7389	0.7405	0.7422	0.7438	0.7454	0.7470	0.7486	0.7502	0.7517	0.7533	0.7549	0.7565
0.7	0.7580	0.7596	0.7611	0.7627	0.7642	0.7658	0.7673	0.7688	0.7704	0.7719	0.7734	0.7749	0.7764	0.7779	0.7794	0.7808	0.7823	0.7838	0.7852	0.7867
0.8	0.7881	0.7896	0.7910	0.7925	0.7939	0.7953	0.7967	0.7981	0.7995	0.8009	0.8023	0.8037	0.8051	0.8065	0.8078	0.8092	0.8106	0.8119	0.8133	0.8146
0.9	0.8159	0.8173	0.8186	0.8199	0.8212	0.8225	0.8238	0.8251	0.8264	0.8277	0.8289	0.8302	0.8315	0.8327	0.8340	0.8352	0.8365	0.8377	0.8389	0.8401
1.0	0.8413	0.8426	0.8438	0.8449	0.8461	0.8473	0.8485	0.8497	0.8508	0.8520	0.8531	0.8543	0.8554	0.8566	0.8577	0.8588	0.8599	0.8610	0.8621	0.8632
1.1	0.8643	0.8654	0.8665	0.8676	0.8686	0.8697	0.8708	0.8718	0.8729	0.8739	0.8749	0.8760	0.8770	0.8780	0.8790	0.8800	0.8810	0.8820	0.8830	0.8840
1.2	0.8849	0.8859	0.8869	0.8878	0.8888	0.8897	0.8907	0.8916	0.8925	0.8934	0.8944	0.8953	0.8962	0.8971	0.8980	0.8988	0.8997	0.9006	0.9015	0.9023
1.3	0.9032	0.9041	0.9049	0.9057	0.9066	0.9074	0.9082	0.9091	0.9099	0.9107	0.9115	0.9123	0.9131	0.9139	0.9147	0.9154	0.9162	0.9170	0.9177	0.9185
1.4	0.9192	0.9200	0.9207	0.9215	0.9222	0.9229	0.9236	0.9244	0.9251	0.9258	0.9265	0.9272	0.9279	0.9285	0.9292	0.9299	0.9306	0.9312	0.9319	0.9325
1.5	0.9332	0.9338	0.9345	0.9351	0.9357	0.9364	0.9370	0.9376	0.9382	0.9388	0.9394	0.9400	0.9406	0.9412	0.9418	0.9424	0.9429	0.9435	0.9441	0.9446
1.6	0.9452	0.9458	0.9463	0.9468	0.9474	0.9479	0.9484	0.9490	0.9495	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9540	0.9545	0.9550
1.7	0.9554	0.9559	0.9564	0.9568	0.9573	0.9577	0.9582	0.9586	0.9591	0.9595	0.9599	0.9604	0.9608	0.9612	0.9616	0.9621	0.9625	0.9629	0.9633	0.9637
1.8	0.9641	0.9645	0.9649	0.9652	0.9656	0.9660	0.9664	0.9667	0.9671	0.9675	0.9678	0.9682	0.9686	0.9689	0.9693	0.9696	0.9699	0.9703	0.9706	0.9710
1.9	0.9713	0.9716	0.9719	0.9723	0.9726	0.9729	0.9732	0.9735	0.9738	0.9741	0.9744	0.9747	0.9750	0.9753	0.9756	0.9759	0.9761	0.9764	0.9767	0.9770
2.0	0.9772	0.9775	0.9778	0.9780	0.9783	0.9786	0.9788	0.9791	0.9793	0.9796	0.9798	0.9801	0.9803	0.9805	0.9808	0.9810	0.9812	0.9815	0.9817	0.9819
2.1	0.9821	0.9824	0.9826	0.9828	0.9830	0.9832	0.9834	0.9836	0.9838	0.9840	0.9842	0.9844	0.9846	0.9848	0.9850	0.9852	0.9854	0.9856	0.9857	0.9859
2.2	0.9861	0.9863	0.9864	0.9866	0.9868	0.9870	0.9871	0.9873	0.9875	0.9876	0.9878	0.9879	0.9881	0.9882	0.9884	0.9885	0.9887	0.9888	0.9890	0.9891
2.3	0.9893	0.9894	0.9896	0.9897	0.9898	0.9900	0.9901	0.9902	0.9904	0.9905	0.9906	0.9907	0.9909	0.9910	0.9911	0.9912	0.9913	0.9915	0.9916	0.9917
2.4	0.9918	0.9919	0.9920	0.9921	0.9922	0.9923	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931	0.9931	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937
2.5	0.9938	0.9939	0.9940	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9945	0.9946	0.9947	0.9948	0.9948	0.9949	0.9950	0.9951	0.9951	0.9952	0.9953
2.6	0.9953	0.9954	0.9955	0.9955	0.9956	0.9957	0.9957	0.9958	0.9959	0.9959	0.9960	0.9960	0.9961	0.9962	0.9962	0.9963	0.9963	0.9964	0.9964	0.9965
2.7	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9970	0.9971	0.9971	0.9972	0.9972	0.9972	0.9973	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9975	0.9976	0.9976	0.9976	0.9977	0.9977	0.9977	0.9978	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9984	0.9985	0.9985	0.9985	0.9985	0.9986	0.9986	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9987	0.9987	0.9988	0.9988	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990	0.9990
3.1	0.9990	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial probability distribution $C_{n,r}\ p^r\ (1-p)^{(n-r)}$

		p																			
n	r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
5	0	0.951	0.774	0.590	0.444	0.328	0.237	0.168	0.116	0.078	0.050	0.031	0.018	0.010	0.005	0.002	0.001	0.000	0.000	0.000	0.000
5	1	0.048	0.204	0.328	0.392	0.410	0.396	0.360	0.312	0.259	0.206	0.156	0.113	0.077	0.049	0.028	0.015	0.006	0.002	0.000	0.000
5	2	0.001	0.021	0.073	0.138	0.205	0.264	0.309	0.336	0.346	0.337	0.313	0.276	0.230	0.181	0.132	0.088	0.051	0.024	0.008	0.001
5	3	0.000	0.001	0.008	0.024	0.051	0.088	0.132	0.181	0.230	0.276	0.313	0.337	0.346	0.336	0.309	0.264	0.205	0.138	0.073	0.021
5	4	0.000	0.000	0.000	0.002	0.006	0.015	0.028	0.049	0.077	0.113	0.156	0.206	0.259	0.312	0.360	0.396	0.410	0.392	0.328	0.204
5	5	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.010	0.018	0.031	0.050	0.078	0.116	0.168	0.237	0.328	0.444	0.590	0.774
10	0	0.904	0.599	0.349	0.197	0.107	0.056	0.028	0.013	0.006	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	1	0.091	0.315	0.387	0.347	0.268	0.188	0.121	0.072	0.040	0.021	0.010	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
10	2	0.004	0.075	0.194	0.276	0.302	0.282	0.233	0.176	0.121	0.076	0.044	0.023	0.011	0.004	0.001	0.000	0.000	0.000	0.000	0.000
10	3	0.000	0.010	0.057	0.130	0.201	0.250	0.267	0.252	0.215	0.166	0.117	0.075	0.042	0.021	0.009	0.003	0.001	0.000	0.000	0.000
10	4	0.000	0.001	0.011	0.040	0.088	0.146	0.200	0.238	0.251	0.238	0.205	0.160	0.111	0.069	0.037	0.016	0.006	0.001	0.000	0.000
10	5	0.000	0.000	0.001	0.008	0.026	0.058	0.103	0.154	0.201	0.234	0.246	0.234	0.201	0.154	0.103	0.058	0.026	0.008	0.001	0.000
10	6	0.000	0.000	0.000	0.001	0.006	0.016	0.037	0.069	0.111	0.160	0.205	0.238	0.251	0.238	0.200	0.146	0.088	0.040	0.011	0.001
10	7	0.000	0.000	0.000	0.000	0.001	0.003	0.009	0.021	0.042	0.075	0.117	0.166	0.215	0.252	0.267	0.250	0.201	0.130	0.057	0.010
10	8	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.023	0.044	0.076	0.121	0.176	0.233	0.282	0.302	0.276	0.194	0.075
10	9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.010	0.021	0.040	0.072	0.121	0.188	0.268	0.347	0.387	0.315
10	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.013	0.028	0.056	0.107	0.197	0.349	0.599
20	0	0.818	0.358	0.122	0.039	0.012	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	1	0.165	0.377	0.270	0.137	0.058	0.021	0.007	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	2	0.016	0.189	0.285	0.229	0.137	0.067	0.028	0.010	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	3	0.001	0.060	0.190	0.243	0.205	0.134	0.072	0.032	0.012	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	4	0.000	0.013	0.090	0.182	0.218	0.190	0.130	0.074	0.035	0.014	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	5	0.000	0.002	0.032	0.103	0.175	0.202	0.179	0.127	0.075	0.036	0.015	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	6	0.000	0.000	0.009	0.045	0.109	0.169	0.192	0.171	0.124	0.075	0.037	0.015	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000
20	7	0.000	0.000	0.002	0.016	0.055	0.112	0.164	0.184	0.166	0.122	0.074	0.037	0.015	0.004	0.001	0.000	0.000	0.000	0.000	0.000
20	8	0.000	0.000	0.000	0.005	0.022	0.061	0.114	0.161	0.180	0.162	0.120	0.073	0.035	0.014	0.004	0.001	0.000	0.000	0.000	0.000
20	9	0.000	0.000	0.000	0.001	0.007	0.027	0.065	0.116	0.160	0.177	0.160	0.119	0.071	0.034	0.012	0.003	0.000	0.000	0.000	0.000
20	10	0.000	0.000	0.000	0.000	0.002	0.010	0.031	0.069	0.117	0.159	0.176	0.159	0.117	0.069	0.031	0.010	0.002	0.000	0.000	0.000
20	11	0.000	0.000	0.000	0.000	0.000	0.003	0.012	0.034	0.071	0.119	0.160	0.177	0.160	0.116	0.065	0.027	0.007	0.001	0.000	0.000
20	12	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.014	0.035	0.073	0.120	0.162	0.180	0.161	0.114	0.061	0.022	0.005	0.000	0.000
20	13	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.015	0.037	0.074	0.122	0.166	0.184	0.164	0.112	0.055	0.016	0.002	0.000
20	14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.037	0.075	0.124	0.171	0.192	0.169	0.109	0.045	0.009	0.000
20	15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.015	0.036	0.075	0.127	0.179	0.202	0.175	0.103	0.032	0.002
20	16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.014	0.035	0.074	0.130	0.190	0.218	0.182	0.090	0.013
20	17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.012	0.032	0.072	0.134	0.205	0.243	0.190	0.060
20	18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.010	0.028	0.067	0.137	0.229	0.285	0.189
20	19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.007	0.021	0.058	0.137	0.270	0.377
20	20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.012	0.039	0.122	0.358